Exam 2 Rev Answers

1. (a) $\mathbf{r}(t) = \left\langle t^2 - 1, 2t - 2, \ln(t) \right\rangle$.
   (b) $|\mathbf{r}(1)| = 3$.
   (c) $\mathbf{a}(t) = \left\langle 2, 0, -1/t^2 \right\rangle$, $\mathbf{a}(1) = \langle 2, 0, -1 \rangle$.
   (d) The length of the curve is $24 + \ln(5)$.
   (e) $\mathbf{T}(t) = \left\langle \frac{2t^2}{1+2t^2}, \frac{2t}{1+2t^2}, \frac{1}{1+2t^2} \right\rangle$, $\mathbf{T}(1) = \langle 2/3, 2/3, 1/3 \rangle$.
   (f) $\kappa(1) = 2/9$.
2. $\mathbf{T}(\pi/6) = \left\langle 2/5, 2\sqrt{3}/5, -3/5 \right\rangle$
3. $\displaystyle\int_0^{\pi/4} \cos(2t)\hat{\boldsymbol{\imath}} + \sin(2t)\hat{\boldsymbol{\jmath}} + t\hat{\mathbf{k}}\, dt = \frac{1}{2}\hat{\boldsymbol{\imath}} + \frac{1}{2}\hat{\boldsymbol{\jmath}} + \frac{\pi^2}{32}\hat{\mathbf{k}}$.
4. The length of the curve $\mathbf{r}(t)$ is $e - \frac{1}{e}$.
5. $\mathbf{T}(t) = \left\langle \frac{t^2}{t^2+2}, \frac{2t}{t^2+2}, \frac{2}{t^2+2} \right\rangle$, $\mathbf{N}(t) = \left\langle \frac{2t}{t^2+2}, \frac{2-t^2}{t^2+2}, \frac{-2t}{t^2+2} \right\rangle$.
6. $\displaystyle\kappa(t) = \frac{\sqrt{2}}{(1 + \cos^2(t))^{3/2}}$.
7. $\mathbf{v}(1) = \langle 2, 1, 3 \rangle$, $\mathbf{a}(1) = \langle 2, 0, 6 \rangle$, speed $= \sqrt{14}$.
8. $a_T = 0$, $a_N = 1$.
9. $\mathbf{r}(t) = \langle 3\sin(t), 4t, 3\cos(t) \rangle$
   (a) $\mathbf{r}(s) = \langle 3\sin(s/5), 4s/5, 3\cos(s/5) \rangle$.
   (b) $\mathbf{T}(0) = \langle 3/5, 4/5, 0 \rangle$ and $\mathbf{N}(0) = \langle 0, 0, -1 \rangle$
   (c) The equation of the normal plane at the point $(0, 0, 3)$ is $3x + 4y = 0$.
10. $\mathbf{a}(t) = \left\langle t, t^2, \cos(2t) \right\rangle$, with initial velocity $\mathbf{v}(0) = \langle 1, 0, 1 \rangle$ and initial position $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$
    (a) $\mathbf{v}(t) = \left\langle 1 + \frac{t^2}{2}, \frac{t^3}{3}, 1 + \frac{\sin(2t)}{2} \right\rangle$.
    (b) $\mathbf{r}(t) = \left\langle t + \frac{t^3}{6}, 1 + \frac{t^4}{12}, \frac{1}{2} + t - \frac{\cos^2(t)}{2} \right\rangle$.