Exam 2 Review

1. An object's velocity $\mathbf{v}$ (in meters per second) at time t seconds is given by $\mathbf{v}(t) = 2t\hat{\imath} + 2\hat{\jmath} + \frac{1}{t}\hat{\mathbf{k}}$, for $t > 0$ seconds. Using this function do the problems below.
   (a) Find the position function $\mathbf{r}(t)$, given that $\mathbf{r}(1) = \mathbf{0}$.
   (b) Find the objects speed at $t = 1$ second.
   (c) Find the acceleration function $\mathbf{a}(t)$, and the acceleration vector at $t = 1$ second.
   (d) How far along the curve does the object travel between 1 and 5 seconds; i.e. what is the arc-length of the curve over $1 \leq t \leq 5$?
   (e) Find the unit tangent vector function $\mathbf{T}(t)$, then find the unit tangent vector at $t = 1$.
   (f) Find the curvature κ of the curve at $t = 1$ second.
2. Find the unit tangent vector $\mathbf{T}$ for the vector position function $\mathbf{r}(t) = \langle t, 2\sin(t), 3\cos(t)\rangle$ at $t = \pi/6$.
3. Evaluate the integral $\displaystyle\int_0^{\pi/4} \cos(2t)\hat{\imath} + \sin(2t)\hat{\jmath} + t\hat{\mathbf{k}}\, dt$.
4. Find the length of the curve $\mathbf{r}(t) = \langle t\sqrt{2}, e^t, e^{-t}\rangle,\ 0 \leq t \leq 1$.
5. Find the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$ for the curve $\mathbf{r}(t) = \langle \frac{1}{3}t^3, t^2, 2t\rangle$.
6. Find the curvature of the curve $\mathbf{r}(t) = \langle \sin(t), \cos(t), \sin(t)\rangle$.
7. Find the velocity, acceleration and speed of the particle with position function $\mathbf{r}(t) = \langle t^2, t, t^3\rangle$ at $t = 1$.
8. Find the tangential and normal components of the acceleration vector if $\mathbf{r}(t) = \cos(t)\hat{\imath} + \sin(t)\hat{\jmath} + t\hat{\mathbf{k}}$.
9. Let $\mathbf{r}(t) = \langle 3\sin(t), 4t, 3\cos(t)\rangle$ be a vector valued function which describes a curve.
   (a) Reparametrize $\mathbf{r}$ in terms of arclength s.
   (b) Find $\mathbf{T}$ and $\mathbf{N}$ (the unit tangent and normal vectors respectively) to the curve at the point $(0, 0, 3)$.
   (c) Find the equation of the normal plane at the point $(0, 0, 3)$.
10. Given the accelleration of a particle $\mathbf{a}(t) = \langle t, t^2, \cos(2t)\rangle$, with initial velocity $\mathbf{v}(0) = \langle 1, 0, 1\rangle$ and initial position $\mathbf{r}(0) = \langle 0, 1, 0\rangle$
    (a) Find the velocity $\mathbf{v}(t)$.
    (b) Find the postion $\mathbf{r}(t)$.