Exam 4 Review Answers

1. Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x)\cos(y)\, dy\, dx$.
   Ans: 1

2. Given the iterated integral $\int_0^1 \int_{\sqrt{x}}^1 x\sqrt{y}\, dy\, dx$,
   (a) Sketch the region of integration.
   (b) Evaluate it directly in the given order.
   Ans: $1/11$
   (c) Evaluate it by first reversing the order of integration.
   Ans: $\int_{y=0}^1 \int_{x=0}^{y^2} x\sqrt{y}\, dx\, dy = \frac{1}{11}$

3. Evaluate $\int_0^1 \int_0^{x^2} (x+2y)\, dy\, dx$.
   Ans: $9/20$

4. Evaluate $\iint_D y^3\, dA$, where D is the triangular region with vertices $(0,2)$, $(1,1)$, and $(3,2)$.
   Ans: $\int_1^2 \int_{2-y}^{2y-1} y^3\, dx\, dy = 147/20$

5. Evaluate by changing to polar coordinates $\iint_R x\, dA$, where R is the disk with center at the origin and radius 5.
   Ans: 0

6. Evaluate by converting to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$$

   Ans: $\int_0^{\pi/2}\int_0^1 e^{r^2} r\, dr\, d\theta = \frac{\pi(e-1)}{4}$

7. Find the volume of the region lying under the graph of the function

$$z = f(x,y) = \cos(x^2+y^2)+1$$

   and over the ring of inner radius 1 and outer radius 3 in the xy-plane centered on the origin. Hint: The volume is given by $\iint_D f(x,y)\, dA$ and try integrating in polar coordinates.
   Ans: $\int_0^{2\pi}\int_1^3 (\cos(r^2)+1)r\, dr\, d\theta = \pi(8-\sin(1)+\sin(9))$

8. Given the double integral below, make a reasonably large sketch of the region of integration, then switch the order of integration (making the appropriate changes to the limits of integration) to evaluate the integral.

$$\int_0^3 \int_{y^2}^9 y\cos(x^2)\, dx\, dy$$

   Ans: $\int_0^9 \int_0^{\sqrt{x}} y\cos(x^2)\, dy\, dx = \frac{1}{4}\sin(81)$

9. Write and evaluate a double integral which will give the volume of the solid under the surface $z = 2x+y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.
   Ans: $\int_0^1 \int_{y^3}^{y^2} 2x+y^2\, dx\, dy = \frac{19}{210}$

Exam 4 Review Answers

10. A washer in the form of a circular ring with inner radius a and outer radius b has a density ρ at any point which is inversely proportional to the distance of that point from the center of the washer. Write and evaluate a double integral which gives an expression for the mass of the washer. Note: mass = (density)(area).
Ans: $\int_0^{2\pi}\int_a^b \frac{k}{r}\, r\, dr\, d\theta = 2k\pi(b-a)$, where k is the constant of proportionality

11. Evaluate the iterated integral $\int_{-1}^{1}\int_{y^2}^{1}\int_0^{1-x} dz\, dx\, dy$.
Ans: $\frac{8}{15}$

12. Evaluate $\iiint_E xz\, dV$, where E is the solid tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(1,1,0)$, and $(0,1,1)$.
Ans: $1/120$

13. Evaluate $\iiint_E x\, dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.
Ans: $16\pi/3$

14. Evaluate $\iiint_E x^2\, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z^2 = 4x^2 + 4y^2$.
Ans: $2\pi/5$

15. Evaluate the integral $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2}(x^2+y^2)^{3/2}\, dz\, dy\, dx$ by changing to cylindrical coordinates.
Ans: $8\pi/35$

16. Set up and evaluate a triple integral in spherical coordinates that gives the mass of the solid hemisphere $x^2 + y^2 + z^2 \le a^2$, $z \ge 0$, if its density δ is proportional to distance z from its base ($\delta = kz$, where k is a positive constant).
Ans: mass $= \int_0^{2\pi}\int_0^{\pi/2}\int_0^a (k\rho\cos(\phi))(\rho^2\sin(\phi))\, d\rho\, d\phi\, d\theta = \frac{1}{4}a^4 k\pi$